A Surprising Appearance of  $\pi$  in the Brain

Mathematical Modeling of Visual Cortex Orientation Maps

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### Abstract

The aim of this paper is to excite the reader about a symbiosis of two seemingly disjoint fields: neuroscience and mathematics. Specifically, we begin with a peculiar experimentally-derived observation from the field of neuroscience: "orientation maps" derived by optical imaging of the visual cortex of the mammalian brain, and how a specific property of these maps appears to hover around the mathematical constant  $\pi$ . We then notice interesting geometric patterns that lend naturally to translational and rotational symmetry constraints. From here, we employ these constraints to build mathematical models for these observations. Finally, we use the theory behind these mathematical models to advance our biological understanding.

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## 1 Introduction

This paper explores some mathematical theory behind an observation from the field of neuroscience. We will first build a brief foundation of the neuroscience relevant for this paper, particularly how columns of neurons in the visual cortex of the mammalian brain respond preferentially to light oriented at certain angles. We will then see how from brain recordings, we can create "orientation maps" describing orientation selectivity in different parts of the visual cortex. From the geometry of these observed orientation maps yields an interesting finding: the density of "pinwheels" (singularity points) roughly equals  $\pi$ . Next in Section 2, we will think geometrically about these observed map patterns and deduce symmetry constraints. We will apply these symmetry constraints to mathematical models that can describe the underlying patterns, focusing specifically on Gaussian random fields (GRFs). These models provide robust mathematical theory, and we will employ them to explain the peculiar  $\pi$  observation we started out with. We will conclude with briefly discussing how this relates to the broader notion of using cross-disciplinary tools for tackling tough problems.

### 1.1 Acknowledgements

Thanks to Professor Dami Lee for not only being open to accepting interdisciplinary paper topics but also the support and encouragement over the last three quarters. While I was admittedly on the fence about completing the entire series, I am confident it is a decision I will not regret. Finally, thanks to Logan for his generous support both in office hours and over Discord. Logan was truly the best teaching assistant I had at UW and deserves recognition for his dedication to helping his students succeed.

### 1.2 A very brief primer on neuroscience

This paper assumes the reader is not familiar with any specific details of neuroscience. The neocortex is a core component of biological intelligence as we know it. The neocortex is unique to mammals, and the human neocortex is roughly the size and thickness of a dinner napkin; it is folded and tightly packed in the outer layers of our brains to increase surface area (*Neuroscience. 2nd edition.*, 2001). A neuron is the atomic functional processing unit of the brain. Their primary purpose is to receive input from other neurons, and determine whether or not to fire an action potential spike which can be picked up by yet more neurons connected to the first (*Neuroscience. 2nd edition.*, 2001). A typical cortical neuron can form tens of thousands of synapses (connections) with other neurons, both proximal (nearby) and distal (far away), resulting in a highly interconnected structure (Gerstner, Kistler, Naud, & Paninski, 2014).

Moreover, the cortex is arranged in a columnar fashion, with "columns" of neurons perpendicular to the cortical surface exhibiting similar response properties. The cortex is composed of several layers and these columns vertically span all layers (*Neuroscience. 2nd edition.*, 2001). Thus, we can say that information is roughly processed in a 2D array of individual functional modules, composed of a cluster of neurons connected not only with other neurons in the column, but also making longer-range connections (Kaschube, Schnabel, & Wolf, 2008). The visual part of the brain is called the visual cortex. The **primary visual cortex** (V1) is the first part of the cortex that processes visual input from electrochemical signals from the eye.

### 1.3 Orientation maps

This paper focuses on response properties of columns within the primary visual cortex. Specifically, **orientation columns** in V1 consist of neurons that respond to bars of light oriented at roughly the same angle (Mountcastle, Davies, & Berman, 1957). Experiments recording from neurons in V1 can determine a particular neuron's orientation preference based on the number of spikes it produces, resulting in interestingly organized geometric patterns (Figure 1).<sup>1</sup>

Mathematically, let  $V \subseteq \mathbb{R}^2$  denote the cortical surface, and let  $r_{\theta}(x, y)$  denote the experimentally-measured cortical response from a particular cortical area  $(x, y) \in V$  when the animal is presented with a stimulus of angle  $\theta \in [0, \pi]$ .<sup>2</sup> Suppose in the experiment the angles  $\theta_1, \theta_2, \ldots, \theta_K$  are used. We can then form a mapping  $f : V \to \mathbb{C}$  from the cortical surface to complex numbers:

$$f(x,y) = \sum_{n=1}^{K} e^{i \cdot 2\theta_n} \cdot r_{\theta_k}(x,y).$$
(1)

Note that this mapping not only encodes the overall orientation preference of an area (as the dominant orientation will exhibit the largest response r) but its complex modulus (essentially magnitude) encodes the degree of selectiveness (where regions selective to only a particular orientation have higher moduli). Also note that we are doubling  $\theta_n$  because we want the angle to span the entire circle  $[0, 2\pi]$  and not just  $[0, \pi]$ . Hence, the pattern of orientation preferences is denoted:

$$\theta(x,y) = \frac{1}{2} \operatorname{Arg} \left( f(x,y) \right), \tag{2}$$

where arg denotes the principal complex argument (i.e., angle), such that  $\theta: V \to [0, \pi]$ .

<sup>&</sup>lt;sup>1</sup>These geometric patterns are not observed in all species of mammals; notably they are absent in rodents (Afgoustidis, 2015). Though evidence also indicates they developed in parallel species whose common ancestor likely did not exhibit geometric orientation maps, indicating some evolutionary benefits (Afgoustidis, 2015).

<sup>&</sup>lt;sup>2</sup>In practice, a response measure such as frequency of spikes may be used. Also note that "cortical area" is used because some electrical recording methods are based on local activity, which is a reasonable proxy for individual activity since neurons within a column exhibit similar response patterns.



Figure 1: (Adapted from Bosking et al. (1997).) (A) Orientation map from tree shrew produced through optical imaging in visual cortex. Lateral refers to the side and rostral refers to the anterior (front). (B) Enlarged portions of the orientation map. Left two indicate pinwheels, middle indicates a linear region, and right shows quasi-periodic pattern.

### 1.4 Pinwheels and pinwheel density

In Figure 1B above, we can observe points where orientation preferences collide. Essentially, these may be thought of as singular points or points of discontinuities, and are referred to as **pinwheels**. Also note the apparent approximate periodic symmetry in the orientation map; we say it exhibits **quasi-periodicity**. Using this observation, we can denote these approximately-repeated regions **hypercolumns**, with spacing  $\Lambda$  (i.e., the distance between regions that prefer the same orientation, typically in the range of 1 mm (Kaschube et al., 2008)). This measurement is supported by Figure 1 above.

Pinwheels can come in many types, and one way to characterize them is by the use of line integrals<sup>3</sup> (Wolf & Geisel, 2003). If  $\gamma_j$  is an arbitrary closed curve around a pinwheel centered at  $z_j$  (suppose for simplicity that  $\gamma_j$  does not intersect any pinwheels so that  $\theta(\mathbf{x})$  is continuous for  $\mathbf{x} \in \gamma_j$  except at the start and endpoint, where the value of  $\theta$  makes a jump discontinuity), then we can denote the **topological change around pinwheel** *i* 

$$q_i = \frac{1}{2\pi} \oint_{\gamma_j} \nabla \theta(\mathbf{x}) \cdot d\mathbf{s},\tag{3}$$

where *ds* refers to the arc length and the  $1/2\pi$  is the normalization factor. Note that  $\theta$  as defined in (2) is cyclic, mapping to the real interval  $[0, \pi]$ , and also continuous except at these pinwheels, it follows by the gradient theorem for line integrals that

$$q_i = \frac{1}{2\pi} \cdot n\pi = \frac{n}{2},\tag{4}$$

where *n* is a integer. (This simplification is because  $\nabla \theta$  is continuous on  $\gamma_j$  and hence we can apply the multidimensional analog of the fundamental theorem of calculus. While  $\gamma_j$  is indeed closed, it has a single discontinuity at the endpoint, and this is where the  $n\pi$  difference comes in.)

Geometrically, the sign of  $q_i$  denotes the direction (clockwise or counterclockwise) in

<sup>&</sup>lt;sup>3</sup>Complex analysis, after all, is "a bunch of line integrals over closed curves" -Dami.

which  $\theta$  is wrapped around pinwheel *i* (see the leftmost two figures in Figure 1B). For example, if our curve  $\gamma_j$  encircles pinwheel *i* counterclockwise, then  $q_i$  being positive suggests the orientation preference also increases counterclockwise (as in the bottom left square in Figure 1B). The magnitude of  $q_i$  denotes the number of complete wrappings exist around the pinwheel. For example, if  $q_i = \pm 1/2$ , then each orientation is represented once in the pinwheel (e.g., the leftmost two figures in Figure 1B). In biology, only pinwheels with this lowest possible topological change  $q_i = \pm 1/2$  are exhibited, and this organization is assumed to be a general feature of orientation maps in the visual cortex (Wolf & Geisel, 2003).

The measurement relevant to this paper is the concept of **pinwheel density**, which is the average number of pinwheels per hypercolumn area  $\Lambda^2$  (Kaschube et al., 2008). Interestingly, experimental observations indicate a universal design principle across various species, where the pinwheel density remains relatively constant, approaching an asymptotic value of  $\pi$ . We will investigate this observation by looking at biologically-plausible theoretical models in the next section.

## 2 Geometric modeling

As this paper is concerned with the mathematics behind orientation maps, and not necessarily the neuroscientific debate of the conditions under which such maps develop, this section aims to present a tool that can be used to model such maps. The geometric properties of orientation maps appear to be closely tied with the horizontal wiring of the visual cortex (i.e., neuronal connections between microcolumns) Afgoustidis (2015). Moreover, these theoretical maps appear to exhibit all features of true biological maps (Kaschube et al., 2008). In this paper, we will think of the cortex as an Euclidean plane and impose symmetry restrictions to model V1-like patterns on Euclidean spaces. We will first look at these symmetries and then discuss a statistical modeling tool that abides by such symmetries.

### 2.1 Symmetries in orientation maps

Following from the quasi-periodic nature exhibited in natural orientation maps, we notice two symmetries. First, there appears to be a *translation symmetry*:

$$f(\mathbf{x} + \mathbf{u}) = f(\mathbf{x}),\tag{5}$$

where  $\mathbf{x} = (x, y)$  and  $\mathbf{u} \in \mathbb{R}^2$  is some vector describing the translational symmetry. Second, there appears to be a *rotation symmetry*:

$$f(R\mathbf{x}) = f(\mathbf{x}), \qquad R_{\theta} = \begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix}$$
 (6)

where  $\theta \in [0, 2\pi]$  is some angle describing the rotational symmetry and  $R_{\theta}$  is the corresponding 2 × 2 rotation matrix. These two symmetries are described by the Euclidean group SE(2), "which is the set of transformations of the plane that preserve Euclidean distance and the orientedness of bases" (Afgoustidis, 2015).

Biologically, these symmetries likely arise due to longer-range horizontal connections made between cortical columns. It is also important to note that biologically, these symmetries are not realized perfectly. There appears to be some sort of inherent randomness producing these quasi-periodic maps; this is an important point to consider when we are making mathematical models.

### 2.2 Random fields

A **random variable** is a variable whose value depends on outcomes of random process. For example, the outcome of the roll of a six-sided die can represent a random variable X. The outcome of a random variable need not be real or integer valued; it can also be vector-valued. The **expected value** of a random variable can be thought of as the mean observed value over an arbitrarily large number of trials. For the dice roll example, we can possibly roll the numbers 1 through 6, so the average value over many trials would be 3.5 (mathematically,  $\mathbb{E}[X] = 3.5$ ). The **variance** of a random variable X is defined  $Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$ . Intuitively, it represents a measure of how much the outcome of a random variable deviates from its mean or expected value.

A random field is a collection of random variables  $\{f(\mathbf{x}) : \mathbf{x} \in \mathcal{X}\}$ , each mapping some parameter space<sup>4</sup> to a Euclidean space M (e.g.,  $\mathbb{R}^n$ ). (Adler & Taylor, 2007). To give a physical example, a random field on a sphere can be used to model meterological characteristics of the Earth such as temperature or pressure.

Let  $z(\mathbf{x})$  be some random field. We can characterize z by its spatial **correlation functions** (Wolf & Geisel, 2003):

$$C(\mathbf{x}, \mathbf{y}) = \mathbb{E}[z(\mathbf{x})\overline{f}(\mathbf{y})], \quad C^*(\mathbf{x}, \mathbf{y}) = \mathbb{E}[z(\mathbf{x})z(\mathbf{y})].$$
(7)

An *ensemble* of random fields can likewise be characterized in terms of its correlation functions defined analogously for more inputs (Schnabel, Kaschube, Löwel, & Wolf, 2007). For our model, these correlation functions can be constrained by our symmetry requirements outlined in equations (5) and (6). In other words, we require the random field to be *statistically invarant with respect to translations and rotations*, such that these shifts yield equal probabilities of outcomes.

The requirement of rotational invariance leads to the observation  $\mathbb{E}[z(\mathbf{x})] = 0$  (i.e., that each random variable has zero mean). Moreover, the combination of translational and rotational invariance means that the correlation function only depends on the distance  $r = ||\mathbf{x} - \mathbf{y}||$ ; i.e.,

$$C(\mathbf{x}, \mathbf{y}) = C(\|\mathbf{x} - \mathbf{y}\|) = C(r).$$
(8)

One of the many models that satisfies this requirement is a Gaussian random field.

### 2.3 Gaussian random field

A random variable *X* is said to be **Gaussian** (or **normally distributed**) if it has the probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2}, \quad x \in \mathbb{R}$$
(9)

for some mean value  $\mu$  and variance  $\sigma^2$  (with  $\sigma > 0$ ). Note that (9) is symmetric about  $x = \mu$ , so  $\mathbb{E}[X] = \mu$ . Likewise, elementary calculus by applying the integral definition of expected value yields  $\operatorname{Var}(x) = \sigma^2$ . One important observation about Gaussians is that they are completely determined by their mean  $\mu$  and variance  $\sigma^2$ . We can generalize

<sup>&</sup>lt;sup>4</sup>This parameter space need not be real or integer valued; it may instead take multidimensional values.

this definition yielding **multivariate Gaussian** random variables taking values in higher dimensional space. A **Gaussian random field (GRF)** is a random field where each distribution is a multivariate Gaussian.

Gaussian random fields can be expressed by the quantities

$$\mathbb{E}[z(\mathbf{x})], \quad \mathbb{E}[\overline{z}(\mathbf{x})], \quad \mathbb{E}[z(\mathbf{x})\overline{z}(\mathbf{y})], \quad \mathbb{E}[z(\mathbf{x})z(\mathbf{y})]$$

which completely describe how they operate (Schnabel et al., 2007). Our model must be constrained by translational invariance, so we must have

$$\mathbb{E}[z(\mathbf{x})] = \mathbb{E}[z(\mathbf{0})] \tag{10}$$

and as hinted in (8) the two-point correlation functions depend only on the distance  $\mathbf{r} = \mathbf{y} - \mathbf{x}$ . Thus define

$$C_1(\mathbf{r}) = \mathbb{E}[z(\mathbf{x})\overline{z}(\mathbf{x}+\mathbf{r})], \qquad (11)$$

$$C_2(\mathbf{r}) = \mathbb{E}[z(\mathbf{x})z(\mathbf{x}+\mathbf{r})].$$
(12)

Observe that

$$C_{1}(\mathbf{r}) = \mathbb{E}[z(\mathbf{x})\overline{z}(\mathbf{x} + \mathbf{r})]$$
(Definition)  

$$= \mathbb{E}[z(-\mathbf{x})\overline{z}(-\mathbf{x} + \mathbf{r})]$$
(Inversion symmetry)  

$$= \mathbb{E}[z(-\mathbf{x} - \mathbf{r})\overline{z}(-\mathbf{x})]$$
(Translation symmetry)  

$$= \mathbb{E}[z(\mathbf{x} + \mathbf{r})\overline{z}(\mathbf{x})]$$
(Inversion symmetry)  

$$= \overline{C}_{1}(\mathbf{r})$$
(Definition)

and hence  $C_1(\mathbf{r})$  is real-valued.



Figure 2: (From Afgoustidis (2015).) Simulated orientation map using a Gaussian random field. The colors in the plot represent different values of complex argument of the field output. Column spacing (wavelength) is  $\Lambda = \frac{1}{3}$ . Note the similarities to the experimentally-derived biological orientation map in Figure 1.

### 2.4 Central Limit Theorem

It is worth briefly mentioning the **Central Limit Theorem** (CLT) to further motivate our Gaussian assumption. The theorem states that given some distribution with mean  $\mu$  and variance  $\sigma^2$ , taking sufficiently large samples from this distribution will lead to these sample means being approximately normally distributed. More specifically, the means of a sample of size *n* distribute normally with mean  $\mu$  and variance  $\sigma^2/n$ , independent of the underlying distribution (we only need each sample to be independent and identically distributed—i.i.d.). One example of this is human heights: plotting the heights of, say, the University of Washington student population will approximately yield a bell curve.

Why is the Gaussian limiting distribution particularly useful in practice? Biological systems are complex, and we seek to understand how larger-scale patterns (e.g., orientation maps) can arise from the aggregation of small-scale processes (e.g., individual neuron orientation preferences) (Frank, 2009). Assuming individual neurons have some common underlying distribution, by the CLT the aggregation of many of them can be approximated by some Gaussian distribution. For reference, there are approximately 140 million neurons in the adult human primary visual cortex (V1), which further provides grounding for our CLT application (Leuba & Kraftsik, 2004).

#### 2.5 Pinwheel density in a GRF

Note that the pinwheel centers are precisely the zeros of our random field  $z(\mathbf{x})$ , as these can be thought of, in essence, as the points of singularity. Then the number of pinwheels in an area A is the number N of zeros in that area. This can be expressed as a integral using the Dirac delta function and the Jacobian of the random field (Wolf & Geisel, 2003)

$$N = \iint_{A} \delta(z(\mathbf{x})) \cdot |J(z(\mathbf{x}))| d^{2}\mathbf{x}$$
(13)

where

$$|J(z(\mathbf{x}))| = \det \begin{pmatrix} \partial_{x_1} \operatorname{Re}(z(\mathbf{x})) & \partial_{x_2} \operatorname{Re}(z(\mathbf{x})) \\ \partial_{x_1} \operatorname{Im}(z(\mathbf{x})) & \partial_{x_2} \operatorname{Im}(z(\mathbf{x})) \end{pmatrix}$$

$$= \partial_{x_1} \operatorname{Re}(z(\mathbf{x})) \partial_{x_2} \operatorname{Im}(z(\mathbf{x})) - \partial_{x_2} \operatorname{Re}(z(\mathbf{x})) \partial_{x_1} \operatorname{Im}(z(\mathbf{x}))$$
(14)

denotes the Jacobian of z(x). Passing the number of zeros to expectation, applying the linearity property of expectation, we get

$$\mathbb{E}[N] = \iint_{A} \mathbb{E}\left[\delta(z(\mathbf{x})) \cdot |J(z(\mathbf{x}))|\right] d^{2}\mathbf{x}$$
(15)

and it thus follows that

$$\rho = \mathbb{E}\left[\delta(z(\mathbf{x})) \cdot |J(z(\mathbf{x}))|\right]$$
(16)

is the expected density of the pinwheels (Wolf & Geisel, 2003).

We will now evaluate  $\rho$ , following the procedure outlined in Wolf and Geisel (2003). Note that  $\rho$  depends only on some location **x**, along with the value  $z(\mathbf{x})$  and the corresponding derivatives  $\nabla z(\mathbf{x})$  at **x**. It is then sufficient to know the joint probability density function  $p(z(\mathbf{x}), \nabla z(\mathbf{x}))$ . However, since we are assuming z is Gaussian, this joint PDF must also be Gaussian, so it can be described by the correlation functions composed of the expected values of all 6 possible ways of multiplying together  $z(\mathbf{x}), \overline{z}(\mathbf{x}), \nabla z(\mathbf{x})$  and  $\nabla \overline{z}(\mathbf{x})$  (see Equation (7)). By our symmetry constraints, the only nonzero correlation functions are

$$\begin{split} \mathbb{E}[z(\mathbf{x})\overline{z}(\mathbf{x})] &= c_a, \\ \mathbb{E}[\nabla z(\mathbf{x})\nabla \overline{z}(\mathbf{x})] &= c_q \end{split}$$

which we will denote as  $c_a$  and  $c_g$ , respectively (Wolf & Geisel, 2003). The aforementioned PDF can then be calculated as follows:

$$p(z(\mathbf{x}), \nabla z(\mathbf{x})) = \frac{1}{\pi^3 c_g^2 c_a} \exp\left(-2\frac{\nabla z(\mathbf{x})\nabla \overline{z}(\mathbf{x})}{c_g}\right) \exp\left(-\frac{z(\mathbf{x})\overline{z}(\mathbf{x})}{c_a}\right).$$
 (17)

Plugging (17) back into (16) and evaluating the expectation using its integral definition, we get (Wolf & Geisel, 2003)

$$\rho = \frac{1}{\pi^3 c_g^2 c_a} \cdot \iiint \exp\left(-2\frac{\nabla z(\mathbf{x})\nabla \overline{z}(\mathbf{x})}{c_g}\right) |J(z(\mathbf{x}))| d^4 \nabla z(\mathbf{x}) \\ \cdot \iint \delta(z) \exp\left(-\frac{z(\mathbf{x})\overline{z}(\mathbf{x})}{c_a}\right) d^2 z(\mathbf{x}).$$
(18)

Then, converting to spherical coordinates with  $g \in [0, \infty)$ ,  $\theta \in [0, \pi)$ , and  $\phi_1, \phi_2 \in [0, 2\pi)$ , we have

$$d^{4}\nabla z(\mathbf{x}) = g^{3}|\cos\theta\sin\theta| \, dg \, d\theta \, d\phi_{1} \, d\phi_{2}$$

as the volume element, so

$$\rho = \frac{1}{\pi^3 c_g^2 c_a} \int_0^\infty g^5 \exp\left(-2\frac{g^2}{c_g}\right) dg \cdot \int_0^\pi |\cos\theta\sin\theta|^2 d\theta \cdot \int_0^{2\pi} \int_0^{2\pi} |\cos\phi_1\sin\phi_2 - \cos\phi_2\sin\phi_1| d\phi_1 d\phi_2$$

$$(19)$$

$$= \frac{c_g}{\pi c_a}$$

$$(20)$$

where in line (20) the integrals were evaluated using WolframAlpha. The next step in this calculation of  $\rho$  comes from expressing  $c_g$  and  $c_a$  as integrals over the power spectral density, which we will not address in this paper. However, applying this procedure (as in Wolf and Geisel (2003) and Schnabel et al. (2007)), it follows that  $\rho$  can be lower bounded by

$$\rho \ge \frac{\pi}{\Lambda^2} (1+\alpha) \ge \frac{\pi}{\Lambda^2},\tag{21}$$

where  $\alpha$  is some nonnegative constant derived from the aforementioned power spectral density. Finally, since  $\Lambda^2$  represents the hypercolumn area (see Section 1.4), it follows that we can lower bound the expected number of pinwheels per hypercolumn area by  $\pi$ .

The error for this bound comes from random deviations in the distribution. Interestingly, for many species the approximate mean pinwheel density is equal to this bound. For example, experimental values include 3.12 in tree shrew, 3.15 in galago (bush baby), and 3.15 in ferret (Kaschube et al., 2010).

# 3 Conclusion

In this paper we have explored mathematical theory behind the modeling of orientation columns in the mammalian visual cortex. Importantly, these theoretically predicted maps (see Figure 2) exhibit all the essential geometric features of biological maps (see Figure 1). We have seen how the use of complex numbers can aid in the modeling of these maps, and how their quasi-periodicity, represented by rotational and translational symmetry constraints, appears to be the driving force behind their beautiful geometric nature. These constraints can be realized in mathematical models; specifically we have seen how the Gaussian random field model naturally yields to theoretical predictions. The mathematical theory behind this model can provide explanations for biologically observed features, such as the density of the pinwheels.

According to Gabriel Silva from the University of California at San Diego,

There is a large intellectual void in our theoretical understanding of many aspects about how the brain works and how it processes information despite ever accumulating volumes of experimental data. A new approach for dealing with such data is needed (Silva, 2011).

While this may be a niche problem that arguably may not fundamentally advance our understanding of the brain, it is hopefully an example of an interdisciplinary approach to solving a problem. Neuroscience is a field with a lot of data and not a lot of theory to explain these data, and using developed tools from fields like mathematics can have a fundamental impact on our ability to understand the brain.

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